

Paramagnetic Reentrance Effect in NS Proximity Cylinders

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Abstract

A scenario for the unusual paramagnetic reentrance behavior at ultra-low temperatures in Nb-Ag, Nb-Au, and Nb-Cu cylinders is presented. For the diamagnetic response down to temperatures of the order 15 mK, the standard theory (quasi-classical approximation) for superconductors appears to work very well, assuming that Ag, Au, and Cu remain in the normal state except for the proximity-induced superconductivity. Here it is proposed that these noble metals may become p-wave superconductors with a transition temperature of order 10 mK. Below this temperature, p-wave triplet superconductivity emerges around the periphery of the cylinder. The diamagnetic current flowing in the periphery is compensated by a quantized paramagnetic current in the opposite direction, thus providing a simple explanation for the observed increase in the susceptibility at ultra-low temperatures.

In 1990 Visani *et al.* [1] reported a surprising paramagnetic reentrance phenomenon at ultra-low temperatures. When a Nb cylinder of diameter $\sim 20\text{-}100\ \mu\text{m}$, covered with a thin film of Ag with a thickness of a few μm , is cooled below the superconducting transition temperature of Nb, the systems initially exhibits the expected diamagnetic response down to temperatures around 10 mK. However, when the temperature is further lowered, the uniform magnetic susceptibility starts to increase again, indicating a decreasing diamagnetic response

at ultra-low temperatures as $T \rightarrow 0$. A very similar observation was later reported for an analogous Nb-Cu system [2,3], and more recently also for a Nb-Au proximity cylinder [4].

The standard theory for the proximity effect is most conveniently described in terms of the quasi-classical approximation, which is a more refined version of the approach first discussed in Ref. [5]. Within this approach, one can describe the diamagnetic response of an S-N system down to 100 mK perfectly well with only one adjustable parameter, the quasi-particle mean free path in the normal state [3,6].

The sudden failure of this quasi-classical approach below 100 mK, suggested by the observed reentrance behavior, implies that some crucial and new element is missing from the usual model. Therefore, Bruder and Imry [7] have proposed a new kind of persistent current around the edge of the normal metal, circulating in the direction opposite to the diamagnetic current [7]. Although this current is associated with an extended state in the weak localization theory, it turns out from a simple estimate that it is of the order of 10^{-3} smaller than the one required to accurately describe the experiments. More recently, Fauchère *et al.* [8] have proposed that the pairing interaction in noble metals, such as Cu, Ag, or Au, is repulsive. This implies that the sign of $\Delta(r)$ changes at the N-S boundary, thus generating an intrinsic π -junction at the boundary. As in the model by Bruder and Imry, this π -junction could generate a current in the direction opposite to the diamagnetic current, resulting in a paramagnetic reentrance effect at ultra-low temperatures. However, a Stoner-analysis suggests that the repulsive interaction used by these authors is likely to cause a magnetic or charge-density-wave instability with a transition temperature of $T_c^M \sim 100$ mK or higher. Other sets of experiments [9] on the proximity effect appear to exclude such a large repulsive potential. Furthermore, if the pairing potential is repulsive, we would rather expect p-wave superconductivity in these noble metals if we follow the analysis of Kohn and Luttinger [10].

Let us therefore propose here that p-wave superconductivity is generated in the outer film below a critical temperature of $T_c \sim 10 - 100$ mK. Earlier experiments [9] have so far not excluded the possibility of anisotropic superconductivity in Cu, Ag, or Au at ultra-low

temperatures. The main problem for the observation of these transitions is that anisotropic superconductors are highly sensitive to disorder. For example, assuming a p-wave transition temperature in the regime of $T_c \simeq 0.1K$, a sample with a quasi-particle mean free path of $10 \mu\text{m}$ or longer would be needed. In our proposed scenario for the NS proximity cylinders, an additional order parameter $\Delta_p(r)$, associated with intrinsic p-wave superconductivity in the outer film, has to establish itself below T_c against the presence of the proximity-generated s-wave superconductivity with $\Delta_s(r)$ penetrating into the outer film. The p-wave superconducting ordering will thus generate a counter-current, reducing the kinetic energy associated with $\Delta_p(r)$. This counter-current will be quantized, and an approximate expression can be derived by minimizing the kinetic energy, as we will show here.

We assume that the London penetration depth of the thin film is larger than d_N , the thickness of the film. The kinetic energy associated with the p-wave superconductor is then approximately given by

$$E_{kin} = \frac{1}{2} \rho_S^p (2eBr - \frac{2\pi n}{l})^2. \quad (1)$$

Here ρ_S^p is the superfluid density of the p-wave superconductor, n is the integer quantum number of the quantized current, and l is the circumference of the thin film, encircling the inner s-wave superconductor. By minimizing with respect to n we find [11]

$$n = 2eB(l/(2\pi))^2 = 0.7958Bl^2, \quad (2)$$

where B and l are expressed in gauss and μm respectively. Hence it is very likely that a spontaneous counter-current with $n = 1, 2, 3, \dots$ is generated, compatible with the actual experimental conditions [1–4]. In deriving Eq. 2 we used $r \simeq l/(2\pi)$, and $d_N \ll l$.

The spatial variation of the magnetic field $B(r)$ is obtained from

$$B_e - B(r) = \frac{1}{\lambda_p^2} \int_r^{r_0} dr' \left(A_\phi(r') - \frac{n}{\phi_0 l} (r_0 - r') \right), \quad (3)$$

where $A_\phi(r) = \int_0^r dr' B(r')$ is the azimuthal component of the vector potential. This leads to a simple differential equation,

$$\frac{\partial^2 B(r)}{\partial r^2} = \frac{1}{\lambda_p^2} B(r), \quad (4)$$

where $\lambda_p^{-2} = \rho_s^p 4\pi e^2 / m$ is the magnetic penetration depth, and ρ_s^p is the superfluid density of the p-wave superconductor. The solution

$$B(r) = B_e \exp [-(r_0 - r)/\lambda_p] + \frac{n}{\lambda_p^2 \phi_0 l} (r - r_0) \quad (5)$$

is valid in the outer region $r_0 - r \leq d_N$. $B(r)$ is exponentially suppressed in the inner Nb-cylinder, as shown in Fig. 1.

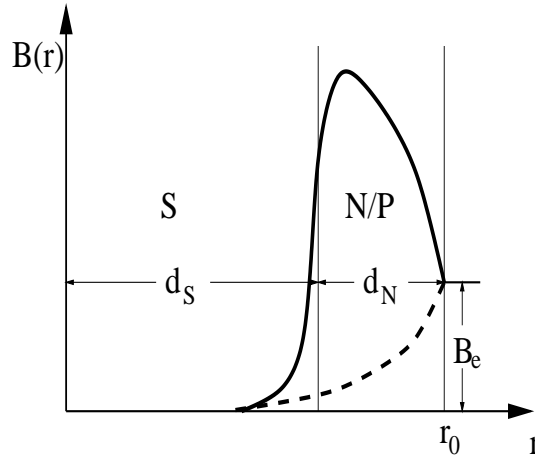


FIG. 1. Sketch of the spatial dependence of the magnetic field in a normal/s-wave (NS) or p-wave/s-wave (PS) proximity cylinder. If there is p-wave superconductivity in the outer layer, the dominant magnetic response is paramagnetic (solid line), with a maximum at the PS interface. The much smaller diamagnetic response is confined to the region close to the surface of the device.

In recent experiments [1–4] it was found that the onset temperature T^* of this paramagnetic reentrance behavior appears to be inversely proportional to the length of the cylinder periphery. As shown in Ref. [1–4], the experimental data can be represented by

$$\chi_r(T) = A \exp \left(-\frac{T}{T^*} \right), \quad (6)$$

where $Al^2 = \text{const.}$ and

$$T^* = \frac{\hbar v_F}{2\pi k_B l}. \quad (7)$$

Let us first attempt to understand the zero-temperature limit of Eq. 6, i.e. the dependence of the prefactor A on the system dimension l . In the absence of a paramagnetic current, the magnetic field is almost completely pushed out of the sample except near the edge of the thin outer film (dashed line in Fig. 1). Once this layer becomes a p-wave superconductor at ultra-low temperatures, the paramagnetic counter current at the PS interface changes $B(r)$ as indicated by the solid line in Fig. 1. Assuming $d_N \ll d_S$, the magnetic field in the sample may then be approximately be expressed as

$$\bar{B} \simeq \frac{n\phi_0}{\pi(l/2\pi)^2} = 4\pi n\phi_0 l^{-2}, \quad (8)$$

where n is a small integer. Under these conditions we can expect that the corresponding susceptibility is given by

$$\chi_r(T=0) = 4\pi n\phi_0 (Bl^2)^{-1}, \quad (9)$$

in agreement with the experiments, and thus χ_r diverges as B^{-1} . Damping effects may smooth out this divergence as $B \rightarrow \frac{B}{B_0^2 + B^2}$, consistent with Fig. 3 in Ref. [3].

At finite temperatures, it appears that thermal phase fluctuations are not negligible anymore. Let us first recall that the superfluid density in the Ginzburg-Landau region is given by $\rho_S^p \sim |\Delta_p|^2$, where Δ_p is the superconducting order parameter of the p-wave superconductor. Since we are considering a quantized flux around the cylinder, the phase coherence along the periphery becomes of crucial importance. Taking into account the possible loss of phase coherence, let us replace $|\Delta_p|^2$ by $|\Delta_p|^2 \langle \exp(i\phi(l) - i\phi(0)) \rangle$ with

$$\langle \exp(i\phi(l) - i\phi(0)) \rangle = \exp\left[-\frac{1}{2} \langle (\phi(l) - \phi(0))^2 \rangle\right]. \quad (10)$$

The average $\langle (\phi(l) - \phi(0))^2 \rangle$ may be evaluated within the one-dimensional model along the azimuthal direction of the cylinder as

$$\langle (\phi(l) - \phi(0))^2 \rangle = \frac{2T}{N(0)} \int \frac{dq}{2\pi} \frac{1 - \cos(ql)}{\xi_0^2 q^2} \simeq \frac{Tl}{N(0)\xi_0^2}, \quad (11)$$

for $T < T_c$ and $\xi_0^2 = \frac{7\xi(3)v_F^2}{2(4\pi T_c)^2}$. These azimuthal fluctuations along the periphery of the cylinder destabilize the diamagnetic response in favor of the paramagnetic counter-current at low temperatures.

Taking into account the length L of the cylinder, this result can be substituted into the expression for ρ_S^p . It is then found that the superfluid density of the p-wave superconductor reduces to

$$\rho_S^p \rightarrow \rho_S^p \exp\left(-\frac{TlL}{N(0)\xi_0^3}\right) = \rho_S^p \exp\left(-\frac{T}{T^*}\right) \quad (12)$$

due to the phase fluctuations. Perpendicular fluctuations along the cylinder are neglected in this context because they play a subdominant role in stabilizing the counter-current along the PS interface. [12]

Hence we can offer an explanation for the observed T - and l -dependence of the exponent, as suggested by the experiments (Eq. 6). In particular, the above expression for the superfluid density implies that

$$T^* = \frac{N(0)\xi_0^3}{lL} = \frac{mp_F\xi_0^3}{2\pi^2lL}, \quad (13)$$

if the density of states $N(0)$ for a 3D system is used. Here m is the quasiparticle mass. Within this approach, T^* exhibits the observed l -dependence (Eq. 6). However, the numerical value which is obtained for T^* is still much larger than the experimentally one, $T^* \approx \frac{v_F}{2\pi k_B l}$. This fact may be remedied by considering a 2D density of states $N(0)$ instead, normalized by the width d_N of the periphery: $N(0)_{2D} = \frac{m}{2\pi d_N}$.

In addition, other possible fluctuations should be considered which may reduce T^* even further. [13] In quasi-one-dimensional systems, phase coherence can be broken by thermal excitations of vortex pairs or phase slip centers [14]. If the spatial extension of the phase slip centers is of the order of ξ with $\xi = \frac{v_F}{2\pi k_B T}$, it is perhaps plausible to have a factor $\exp(-l/\xi)$, as observed in the experiments, since the phase slip centers cannot be densely populated. In any case, a quantitatively correct interpretation of the temperature-dependence in the exponential factor appears to be difficult to find.

In conclusion, we propose (1) that noble metals may become p-wave superconductors with $T_c \sim 10 - 100\text{mK}$. (2) With this assumption, the paramagnetic reentrance behavior at ultra-low temperatures can be described in a quantitative way. (3) Therefore this behavior

should not extend beyond noble metals. More experiments with Pt, Ir, and Os would be of great interest.

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